

# Event Studies Without Market Expectations

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Under standard assumptions, the average price change of a security caused by a materially important announcement is zero. Event studies, used to test whether announcements are materially important, must therefore “bin” announcements into above or below market expectations. Otherwise, the approach would have no power to reject a null hypothesis of a no announcement effect. In situations where market expectations are unknown, we show a Wald statistic (the square of the standardized cumulative abnormal return found in standard event studies) is an easily implementable and powerful approach to test whether an event affects securities prices. We also provide three examples of its applicability.

**keywords:** Event study, Wald statistic, efficient market hypothesis.

## 1 INTRODUCTION

Event studies play an important role in the study of how different phenomena affect firms. Specifically, if a certain type of event happens, then how is a firms' profitability (as measured by their stock price) affected? The events studied range widely from whether stock splits (Dolley (1933)), earnings announcements (Prakash (2013)), or merger and acquisitions (Keown and Pinkerton (1981)) affect prices to whether Twitter sentiment (Ranco et al. (2015)), political events (Herron (2000)) or data breaches (Johnson et al. (2017)) are materially important. In the analysis below, we show how our extension of the standard event study analysis is important to three distinctly different types of

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events: earnings announcements, winning the right to host the Olympic Games, and patent related judicial decisions.

In general, events studies are based on a simple model of security prices that states the price of a security changes if a materially important event occurs and is announced to the market. If a large number of independent and materially important announcements occur, then one could also posit the average change in the price of the underlying security(ies) would be zero.<sup>1</sup> This is the test the canonical event study implements (refer to MacKinlay (1997) for a well-cited discussion) when testing whether a type of announcement is materially important. Specifically, an event study uses multiple independent announcements and calculates the average price change of the affected securities. If the average is different from zero, then it is deemed the event is materially important. However, the unconditional average is zero whether it is materially important or not. In other words, the standard statistical approach used in an event study analysis has no power in stating the event is materially important unless market expectations are known. Given this shortcoming, we provide an approach to test for material important events without knowing market expectations.

In defense of the standard approach, it is powerful if market expectations are known and security price changes are “binned” accordingly. For example, earnings announcements may be above or below market expectations. If many announcements are observed, then the average price change as a result of the announcement would be zero. To circumvent this issue, researchers bin the announcements into categories that reflect above market expectations and below market expectations. For those in the above expectations bin, we see a positive impact. For securities and announcements that reflect below average expectations, we observe a negative impact (refer to MacKinlay (1997) for an example). However, if market expectations are unknown and the above and below market expectations are lumped into one bin, then it is likely the standard event study approach will fail to detect the impact of earnings announcements on securities prices. As previously stated, the standard statistical approach has no power to reject a false null hypothesis. We provide this as one of the applications in Section 3.3.1. Specifically, we fail to reject an earnings announcement when market expectations are unknown or

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<sup>1</sup>To demonstrate it in a basic environment, suppose there is a contract that pays “ $X_g$ ” if the “good” event happens and “ $X_b$ ” if the bad event occurs. The “good” and “bad” outcomes occur with probability  $p$  and  $(1-p)$ , respectively. In a risk-neutral environment, or one where a sufficiently large number of the contracts can be purchased and the outcomes of the events are independent, then the price of the contract prior to realization of the outcome would be

$$price_{pre-event} = pX_g + (1 - p)X_b.$$

After the realization or announcement of the outcome, the price will be equal to the payoff, or  $X_g$  in the case of the good event, or  $X_b$  in the case of the bad event, and on average we would observe after the event the price to be

$$\widehat{price}_{post-event} = pX_g + (1 - p)X_b.$$

Therefore, a test of the material effect of the event on the contracts price using the average of the difference between the price before and after the event, or  $price_{pre-event} - \widehat{price}_{post-event}$ , would fail to reject it is different from zero because it *is* zero on average.

the data isn't separated into above and below market expectations.

To avoid the need for binning, i.e., knowing market expectations, we propose the use of a Wald statistic. The approach is a simple extension of the standard event study analysis. Specifically, for each announcement, the researcher calculates and standardizes abnormal returns in an event window relative to an estimation window using a model of the researcher's choice (e.g., a "market model"). While the standard event study approach bins the observations, sums the abnormal returns, and tests whether the sum is different from zero using a standard normal distribution, our approach using the Wald statistic involves first squaring the abnormal returns, then summing and comparing the statistic to the appropriate ( $\chi^2$ ) distribution. The key is that observations do not average to zero as their contributions are all squared and thus positive.

To detail our proposed method, we first describe the Wald statistic (Section 2) and compare its implementation to the standard statistical approach to event study analysis (Section 2.5). We then analyze its power (Section 3.1) and provide three applications where in the latter case, the null hypothesis under the standard approach would fail to be rejected, while using our method based on the Wald statistic, it is rejected (Section 3.3).

To understand the applicability of our approach and alternative solutions to testing for the material importance of specific announcements when market expectations unknown, it is important to document the related literature. The event study literature is replete with approaches for testing market efficiency using the concept of abnormal returns. For simplicity, we classify them according to the following criteria: (i) those that use the entire distribution of returns and require no parametric assumptions, (ii) those based on normal distribution theory and require knowledge of market expectations, and (iii) those that test for a change in variance. Regardless of the approach, the generic alternate hypothesis is that the event had some effect on the returns. This may be further specified as a change in mean, change in variance, or change in the distribution all together. Note, a brief survey of these methods is described by Serra (2002).

In terms of our first classification, the non-parametric tests encompass a variety of approaches. The stochastic dominance (SD) criteria, as discussed by Falk and Levy (1989), Hertzels (1991), and Larsen and Resnick (1999), use the entire distribution of returns in assessing market efficiency. As a statistical approach, SD is closely aligned with the non-parametric Kolmogorov-Smirnov (K-S) test, which in the one-sample case, tests an empirical cumulative distribution function (CDF) against a reference CDF, or in the two-sample case, compares two empirical CDFs for equality. The distribution would likely be comprised of standardized observations, where the standardization involves the incorporation of risk (e.g., the standard deviation). As an alternative, rank tests such as Corrado (1989) and more recently Kolar and Pynnonen (2011) also provide a non-parametric approach to testing for a material impact of an announcement on a security's price. However, these studies are likely more similar to the standard event study, or our second classification, at least in terms of our analysis, as the test requires binning or market expectations.

In terms of our second classification, or what we loosely call an "event study," our method is built on the work of Patell (1976), or the use of standardized cumulative

abnormal returns, and Brown and Warner (1980) and Boehmer et al. (1991) who adjust for measuring the variance in returns caused by the event. The work is surveyed by MacKinlay (1997), Serra (2002), Kothari and Warner (2007) and Corrado (2011) among others. The event study approach is based on normal distribution theory and to be effective in identifying market inefficiency, requires knowledge of market expectations. Prior to any statistical analysis, observations need to be binned according to above or below market expectations. Normal performance of the return is measured within an estimation window using quite often a “constant mean” or “market” model, both of which require an assumption of normality. The abnormal return can then be estimated as the difference between the observed return in the event window and the normal performance, which under the null hypothesis, should be the same in both estimation and event windows. The abnormal returns may then be averaged over firm and then aggregated over the event window. A test based on a standard normal statistic using the mean cumulative abnormal return is then employed to identify a possible mean effect.

For our final classification, testing for market efficiency by looking for a change in variability is well established in the literature (Beaver (1968); Rohrbach and Chandra (1989); Boehmer et al. (1991)). Beaver defines a  $U$ -statistic, which is the ratio of the squared residual from event window (report period) data to the estimated variance from the estimation window (non-report period) data, as a way to quantify the effect of an earnings announcement. Rohrbach and Chandra compare the power of Beaver’s  $U$  to a statistic they term May’s  $U$ , which is based on the absolute value of the standardized residual. When the residuals are normally distributed, Beaver’s  $U$  is optimal in detecting a change in variance. However, in the presence of leptokurtic residuals, May’s  $U$  dominates Beaver’s  $U$ , although both are unreliable. They also give an empirical distribution approach for testing Beaver’s  $U$  and May’s  $U$  in this circumstance and offer a non-parametric statistical approach based on the ranks of the squared residuals. Boehmer et al. (1991) discuss the importance of incorporating event-induced variance changes in the test for zero average abnormal returns. An underestimation of the variance (that is assuming the variance in the event window is equal to that in the estimation window) can lead to rejection of the null hypothesis more frequently than it should. Furthermore, they propose a standardized cross-sectional test that uses variance information from both estimation and event windows.

There are similarities between our Wald statistic approach and the three methods outlined above. For example, the squared residual in the numerator of Beaver’s  $U$  statistic is equivalent to the square of the abnormal return under the hypothesis that the mean return does not change from estimation window to event window. In other words, without close inspection, it may appear the Wald test we are proposing is equivalent to testing for a change in variance. We are not, as indicated by the fact we estimate different variances for the estimation and event windows.

A variance change can occur in the presence or absence of a change in mean. If one were only testing for a change in variance, estimates of the variability in the estimation and event windows would be derived, relative to the mean in each window. One could then use these estimates in a two-sample  $F$ -test. In the Beaver and Hertzler papers, both authors make the assumption of absence in a change in mean return, when testing for a

change in variance.

In our work, we do not need to make any assumptions about a possible change in variance between the estimation and event windows. Our approach is based on quantifying a change in mean with an estimator that captures the difference between the mean return under the null hypothesis and that observed in the event window. Normalization of the squared residuals is not accomplished by dividing by the variance in the estimation window. Instead, because we are testing for a change in mean, we divide by the variance of the estimator of this change in mean, which results in the Wald statistic. This estimator variance includes measures of variability from both estimation and event windows. To put it different, we build on Patell’s work of standardized residuals, and use Beaver’s approach by squaring them, but we use the proper variance of the estimator to ensure we are testing a change in the mean rather than the variance. To summarize and make clear, our approach may look like a test for a change in variance using standardized cumulative abnormal returns. However, it is a test for a change in the mean.

As our method is more flexible than classification (iii) and is capable of accomplishing the test of an announcements effect even when binning isn’t available (ii), the last comparison is related to (i). In this case, we show empirically in Section 3.1 that our Wald statistic is more powerful than a K-S test based on assumptions following power estimates found in MacKinlay (1997).

As we are proposing a new method to test for materially important events when the market expectations are not known, it is important to consider when such situations occur and when our method would be needed. In Section 3.3, we provide two applications: patent decisions and International Olympics Committee announcements of whether a country has won the right to host the Olympic Games. In both cases, the announcement may sometimes be good or bad. For example, a firm may technically lose a patent case, but the stock price rises because it is better than the market expected. As a result, our method would be needed to see whether patent decisions mattered. However, many other applications beyond our two could be considered. For example, does the retirement of a CEO, a change in greenhouse gas emission standards, or tariffs impact companies’ bottom lines? As these events may be good or bad for companies depending upon the situation, any standard event study could fail to detect their importance unless a method like the one being proposed here is used.

## 2 METHODOLOGY

### 2.1 The Model

In the following discussion, our notation is adopted from MacKinlay (1997). In an event study, we look to determine if the occurrence of the event resulted in some “abnormal” behavior of a certain outcome. If so, then it will be concluded that the announcement of the event had a material impact on the outcome. In general, and in all of our examples, the event date is the date a public announcement is made about an event that may contain material information, i.e., affect a security’s price.

The “event date” is set to be  $\tau = 0$ . We define an event window as

$$\tau_1 \leq \tau < \tau_2 \equiv [\tau_1, \tau_2) , \quad (1)$$

where  $\tau_1$  is the initial day of the event window and  $\tau_2$  is the final (not included) day. Usually,  $\tau_1 \leq 0 < \tau_2$ . To measure any abnormality, an estimation window is used to quantify what is “normal” behavior of the outcome, that is, behavior prior to the event. We define the estimation window as

$$T_0 \leq t < T_1 \equiv [T_0, T_1) , \quad (2)$$

where  $T_0$  is the initial day of the estimation window and  $T_1$  is the final (not included) day. Note  $t$  is used as the index for time points in the estimation window while  $\tau$  is used for time points in the event window. This delineation is useful when computing similar statistical quantities in each window.

A statistical comparison is made between a security’s price in the estimation and event windows. If the difference is statistically significant, then we may conclude the event had a material impact.

We choose to use a simple linear market model, which is arguably the standard for event studies, for the underlying relationship between the asset returns for a firm and the overall market returns. Specifically, assume we have  $t = 1, 2, \dots, N_t$  observations in the estimation window given by  $\{(R_{mt}, R_{it})\}_{t=1}^{N_t}$ , where  $R_{mt}$  is the market return at time  $t$  and  $R_{it}$  is the return for firm  $i$  at time  $t$ . Using the market model, we express the return for firm  $i$  as

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} , \quad (3)$$

where  $\alpha_i$  and  $\beta_i$  are the population intercept and slope of the linear model in the estimation window, and  $\epsilon_{it}$  is the random error term with expected value equal to zero, or  $E(\epsilon_{it}) = 0$ , and variance  $V(\epsilon_{it}) = \sigma_{\epsilon_i}^2$ . As a consequence,  $E(R_{it}) = \alpha_i + \beta_i R_{mt}$  and  $V(R_{it}) = \sigma_{\epsilon_i}^2$ .

Similarly, in the event window, we have  $\tau = 1, 2, \dots, N_\tau$  observations of the form  $\{(R_{m\tau}, R_{i\tau})\}_{\tau=1}^{N_\tau}$ , where  $R_{m\tau}$  is the market return at time  $\tau$  and  $R_{i\tau}$  is the return for firm  $i$  at time  $\tau$ . We express the return for firm  $i$  as

$$R_{i\tau} = \alpha'_i + \beta'_i R_{m\tau} + \epsilon'_{i\tau} , \quad (4)$$

where  $\alpha'_i$  and  $\beta'_i$  are the population intercept and slope of the linear model in the event window, and  $\epsilon'_{i\tau}$  is the random error term. We similarly note that  $E(\epsilon'_{i\tau}) = 0$  and  $V(\epsilon'_{i\tau}) = \sigma_{\epsilon_i}^2$ , which result in  $E(R_{i\tau}) = \alpha'_i + \beta'_i R_{m\tau}$  and  $V(R_{i\tau}) = \sigma_{\epsilon_i}^2$ .

The Wald statistic we propose is testing for a change in the expected return between estimation and event windows, but does not test for a change in the variance. Testing for a change in expected return is equivalent to testing

$$\begin{aligned} H_0 : & \alpha_i = \alpha'_i \text{ and } \beta_i = \beta'_i \\ H_a : & \alpha_i \neq \alpha'_i \text{ or } \beta_i \neq \beta'_i \end{aligned} , \quad (5)$$

whereas a change in variance tests

$$\begin{aligned} H_0 : \sigma_{\epsilon_i}^2 &= \sigma_{\epsilon_i'}^2 \\ H_a : \sigma_{\epsilon_i}^2 &\neq \sigma_{\epsilon_i'}^2 \end{aligned} \quad (6)$$

In the formulation of our Wald statistic, we are allowing the variances in the estimation and event windows to be different.

The reader will note that these formulations can easily accommodate additional factors in both the estimation window and event window models.<sup>2</sup>

## 2.2 Parameter Estimation

The regression parameters for each simple linear model can be estimated using ordinary least squares (OLS). For the estimation window model in Eqn. (3), we find

$$\begin{aligned} \hat{\beta}_i &= \frac{\sum_{t=1}^{N_t} (R_{mt} - \overline{R_m})(R_{it} - \overline{R_i})}{\sum_{t=1}^{N_t} (R_{mt} - \overline{R_m})^2} \\ \hat{\alpha}_i &= \overline{R_i} - \hat{\beta}_i \overline{R_m} \end{aligned} \quad (7)$$

where  $\overline{R_m}$  and  $\overline{R_i}$  are the sample mean returns for the market and firm  $i$ , respectively. Similar expressions hold for the event window model in Eqn. (4)

$$\begin{aligned} \hat{\beta}_i' &= \frac{\sum_{\tau=1}^{N_\tau} (R_{m\tau} - \overline{R_m'})(R_{i\tau} - \overline{R_i'})}{\sum_{\tau=1}^{N_\tau} (R_{m\tau} - \overline{R_m'})^2} \\ \hat{\alpha}_i' &= \overline{R_i'} - \hat{\beta}_i' \overline{R_m'} \end{aligned} \quad (8)$$

where  $\overline{R_m'}$  and  $\overline{R_i'}$  are the sample mean returns for the market and firm  $i$  within the event window, respectively.

Within the estimation window, a point estimate for the mean  $E(R_{it})$  is given by

$$\begin{aligned} \widehat{R}_{it} &= \hat{\alpha}_i + \hat{\beta}_i R_{mt} \\ &= \overline{R_i} + \hat{\beta}_i (R_{mt} - \overline{R_m}) \end{aligned} \quad (9)$$

This equation can also be used to predict the value of a return for firm  $i$  at time  $t = h$  at a new market return value  $R_{mh}$

$$\begin{aligned} \widehat{R}_{ih} &= \hat{\alpha}_i + \hat{\beta}_i R_{mh} \\ &= \overline{R_i} + \hat{\beta}_i (R_{mh} - \overline{R_m}) \end{aligned} \quad (10)$$

<sup>2</sup>Instead of using a simple linear market model, a more complex multiple linear model that includes other factors may be employed. All of the analysis that follows extends to this multiple regression case, however matrix notation must be utilized in most of the derivations. The values of the covariates are specified in a design matrix  $\mathbf{X}$ , which plays a fundamental role in the calculation of the variances for the respective point estimates (refer to Kutner et al. (2005), pp. 206-209 and pp. 214-231 for details).

The statistical properties of our parameter estimates<sup>3</sup>, or  $\widehat{\alpha}_i$  and  $\widehat{\beta}_i$ , allow us to compute the population mean and variance of this point estimate to be

$$\begin{aligned} E(\widehat{R}_{ih}) &= E(\widehat{\alpha}_i) + R_{mh}E(\widehat{\beta}_i) \\ &= \alpha_i + \beta_i R_{mh} \end{aligned} \quad (12)$$

and

$$\begin{aligned} V(\widehat{R}_{ih}) &= V(\overline{R}_i) + (R_{mh} - \overline{R}_m)^2 V(\widehat{\beta}_i) \\ &= \sigma_{\epsilon_i}^2 \left( \frac{1}{N_t} + \frac{(R_{mh} - \overline{R}_m)^2}{TSS_{R_m}} \right), \end{aligned} \quad (13)$$

where  $TSS_{R_m}$  is defined as the total sum of squares of  $R_{mt}$  in the estimation window

$$TSS_{R_m} = \sum_{t=1}^{N_t} (R_{mt} - \overline{R}_m)^2. \quad (14)$$

In general, the population variances  $\sigma_{\epsilon_i}^2$  and  $\sigma_{\epsilon_i'}^2$  will be unknown. In such circumstances, the population variances are replaced by their respective mean square errors (MSE), which are unbiased estimates of the population parameters,

$$\begin{aligned} \widehat{\sigma}_{\epsilon_i}^2 &= \frac{1}{N_t - 2} \sum_{t=1}^{N_t} (R_{it} - \widehat{R}_{it})^2 \\ \widehat{\sigma}_{\epsilon_i'}^2 &= \frac{1}{N_\tau - 2} \sum_{\tau=1}^{N_\tau} (R_{i\tau} - \widehat{R}_{i\tau})^2 \end{aligned} \quad (15)$$

### 2.3 Hypothesis

To test if the event had a material impact on the security's price, we (and the event studies literature more broadly) compute the difference between the actual return observed during the event window,  $R_{i\tau}$ , and the expected return  $E(R_{i\tau})$ , computed under  $H_0$  as defined in Eqn. (5)

$$\begin{aligned} E(R_{i\tau}) &= \alpha'_i + \beta'_i R_{m\tau} \\ &= \alpha_i + \beta_i R_{m\tau} \\ &\approx \widehat{\alpha}_i + \widehat{\beta}_i R_{m\tau} \\ &= \widehat{R}_{i\tau} \end{aligned} \quad (16)$$

If the event had no material impact on the security's price, we would expect the mean return to be the same in both estimation and event windows. The difference is defined

<sup>3</sup>The statistical properties of the estimates of our intercept and slope parameters for the estimation window can easily be shown to be (refer to Kutner et al. (2005), p. 41, 49)

$$\begin{aligned} E(\widehat{\alpha}_i) &= \alpha_i, & V(\widehat{\alpha}_i) &= \sigma_{\epsilon_i}^2 \left( \frac{1}{N_t} + \frac{(\overline{R}_m)^2}{\sum_{t=1}^{N_t} (R_{mt} - \overline{R}_m)^2} \right) = \sigma_{\epsilon_i}^2 \left( \frac{1}{N_t} + \frac{(\overline{R}_m)^2}{TSS_{R_m}} \right), \\ E(\widehat{\beta}_i) &= \beta_i, & V(\widehat{\beta}_i) &= \frac{\sigma_{\epsilon_i}^2}{\sum_{t=1}^{N_t} (R_{mt} - \overline{R}_m)^2} = \frac{\sigma_{\epsilon_i}^2}{TSS_{R_m}}. \end{aligned} \quad (11)$$



here and in the literature as the “abnormal return”  $\widehat{AR}_{i\tau}$ , which is computed using (10) and (16)

$$\begin{aligned}\widehat{AR}_{i\tau} &= R_{i\tau} - E(R_{i\tau}) \\ &\approx R_{i\tau} - \widehat{R}_{i\tau} \\ &= R_{i\tau} - \widehat{\alpha}_i - \widehat{\beta}_i R_{m\tau} \\ &= R_{i\tau} - \overline{R}_i - \widehat{\beta}_i (R_{m\tau} - \overline{R}_m)\end{aligned}\quad , \quad (17)$$

More specifically, we and the event studies literature test the hypothesis

$$\begin{aligned}H_0 : E(\widehat{AR}_i) &= 0 \\ H_a : E(\widehat{AR}_i) &\neq 0\end{aligned}\quad , \quad (18)$$

where the expected value and variance of  $\widehat{AR}_{i\tau}$  are computed from (17) as

$$\begin{aligned}E(\widehat{AR}_{i\tau}) &= E(R_{i\tau}) - E(\widehat{\alpha}_i) - R_{m\tau} E(\widehat{\beta}_i) \\ &= \alpha'_i + \beta'_i R_{m\tau} - \alpha_i - \beta_i R_{m\tau} \\ &= (\alpha'_i - \alpha_i) + (\beta'_i - \beta_i) R_{m\tau}\end{aligned}\quad , \quad (19)$$

and

$$\begin{aligned}V(\widehat{AR}_{i\tau}) &= V(R_{i\tau}) + V(\overline{R}_i) + (R_{m\tau} - \overline{R}_m)^2 V(\widehat{\beta}_i) \\ &= \sigma_{\epsilon_i}^2 + \frac{\sigma_{\epsilon_i}^2}{N_t} + (R_{m\tau} - \overline{R}_m)^2 \frac{\sigma_{\epsilon_i}^2}{TSS_{R_m}} \\ &= \sigma_{\epsilon_i}^2 + \sigma_{\epsilon_i}^2 \left( \frac{1}{N_t} + \frac{(R_{m\tau} - \overline{R}_m)^2}{TSS_{R_m}} \right)\end{aligned}\quad . \quad (20)$$

Note that  $V(\widehat{AR}_{i\tau})^4$  contains contributions from the variability in both the estimation and event windows.

Rather than looking at a single day, it is standard in the event studies literature to aggregate returns over several days, as it is unclear when the market was aware of the potentially material information and how long it took for the information to affect the security’s return. As a result, we aggregate the abnormal returns over the event window by computing the cumulative abnormal return of the observations

$$\widehat{CAR}_i = \sum_{\tau=1}^{N_\tau} \widehat{AR}_{i\tau} \quad , \quad (21)$$

and test the hypothesis

$$\begin{aligned}H_0 : E(\widehat{CAR}_i) &= 0 \\ H_a : E(\widehat{CAR}_i) &\neq 0\end{aligned}\quad . \quad (22)$$

The expected value of  $\widehat{CAR}_i$  is

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<sup>4</sup>This is true because the covariance between the pairs of random terms in Eqn (17) is zero (refer to Kutner et al. (2005), p. 53 for a proof).

$$\begin{aligned}
E(\widehat{CAR}_i) &= \sum_{\tau=1}^{N_\tau} E(\widehat{AR}_{i\tau}) \\
&= \sum_{\tau=1}^{N_\tau} \left( (\alpha'_i - \alpha_i) + (\beta'_i - \beta_i) R_{m\tau} \right) , \\
&= N_\tau \left[ (\alpha'_i - \alpha_i) + (\beta'_i - \beta_i) \overline{R'_m} \right]
\end{aligned} \tag{23}$$

where  $\overline{R'_m}$  is the sample mean return for the market within the event window. Notice that  $E(\widehat{CAR}_i) = 0$  when  $\alpha_i = \alpha'_i$  and  $\beta_i = \beta'_i$ . Thus, the null and alternate hypotheses in Eqn. (22) can be considered as a test on whether the information affects a security's model parameters  $\alpha$  or  $\beta$ , as originally expressed in Eqn. (5).

As a sum of a possibly large number of random variables  $\widehat{AR}_{i\tau}$ , we may conclude by the Central Limit Theorem that  $\widehat{CAR}_i$  is approximately normally distributed where the variance of  $\widehat{CAR}_i$  is

$$\begin{aligned}
V(\widehat{CAR}_i) &= V(\sum_{\tau=1}^{N_\tau} R_{i\tau} - \overline{R}_i - (R_{m\tau} - \overline{R}_m) \hat{\beta}_i) \\
&= N_\tau V(R_{i\tau}) + V(N_\tau \overline{R}_i) + V(\hat{\beta}_i \sum_{\tau=1}^{N_\tau} (R_{m\tau} - \overline{R}_m)) \\
&= N_\tau V(R_{i\tau}) + N_\tau^2 V(\overline{R}_i) + \left( \sum_{\tau=1}^{N_\tau} (R_{m\tau} - \overline{R}_m) \right)^2 V(\hat{\beta}_i) . \\
&= N_\tau \sigma_{\epsilon'_i}^2 + \sigma_{\epsilon_i}^2 \left[ \frac{N_\tau^2}{N_t} + \frac{[\sum_{\tau=1}^{N_\tau} (R_{m\tau} - \overline{R}_m)]^2}{TSS_{R_m}} \right]
\end{aligned} \tag{24}$$

As in Eqn. (20), the second equality arises from the fact that any two pairs of the random terms are uncorrelated.

To reiterate, the Wald test we propose in the next section does not require an assumption that  $\sigma_{\epsilon'_i}^2 = \sigma_{\epsilon_i}^2$ , nor is the Wald statistic used as a test of a change in variance. That being said, it is standard in the event studies literature to assume  $\sigma_{\epsilon'_i}^2 = \sigma_{\epsilon_i}^2$ , and  $N_t$  is sufficiently large so that

$$V(\widehat{CAR}_{i\tau}) = N_\tau \sigma_{\epsilon_i}^2 . \tag{25}$$

This result makes it easy to apply our approach given a standard event study analysis.

## 2.4 Wald test

To this point in our analysis, the market model and estimation procedure we have outlined follows the standard event study analysis. However, event studies do not use a single observation to test whether an event has had a material impact. There isn't sufficient power when an abnormal return is on the order of 0.01 and the standard deviation in the event or estimation window is on the order of 0.02. To improve this lack of power, researchers calculate the average  $\widehat{CAR}_i$  across many of the same types of events. However, as discussed in the Introduction and Section 3.1, the canonical event study test has no power unless the events can be binned into above and below market expectations.

To avoid the binning issue, we use a Wald statistic to test for a cumulative abnormal return at the firm level, or  $H_0 : E(\widehat{CAR}_i) = 0$  versus  $H_a : E(\widehat{CAR}_i) \neq 0$ . In general, a Wald statistic takes the following form

$$\begin{aligned} W &= (\widehat{\theta} - \theta)I(\widehat{\theta})(\widehat{\theta} - \theta) \\ &= \frac{(\widehat{\theta} - \theta)^2}{V(\widehat{\theta})} \end{aligned} \quad (26)$$

where  $\theta$  is the value of the population parameter to be tested,  $\widehat{\theta}$  is the maximum likelihood estimate (MLE) of  $\theta$ , and  $I(\widehat{\theta}) = V(\widehat{\theta})^{-1}$  is the Fisher information, i.e., the inverse of the variance of the estimator  $\widehat{\theta}$ . If  $\widehat{\theta}$  is normally distributed, then the Wald statistic follows a  $\chi^2$ -distribution with  $\nu = 1$  degrees of freedom (df). Otherwise, the Wald statistic asymptotically follows a  $\chi^2$ -distribution. As  $\widehat{CAR}_i$  is normally distributed due to the Central Limit Theorem, we can use the former result. In either case, the Wald test statistic does not require security price returns to be normally distributed.

Using the Wald statistic to test  $H_0$  translates into evaluating whether

$$W_i = \frac{(\widehat{CAR}_i - 0)^2}{V(\widehat{CAR}_i)} \quad (27)$$

is unusually large, where  $\theta = E(\widehat{CAR}_i) = 0$ ,  $\widehat{\theta} = \widehat{CAR}_i$ , and  $V(\widehat{\theta}) = V(\widehat{CAR}_i)$ . If  $\alpha = 0.05$  denotes the significance level and  $W_i > \chi_{0.05}^2(1)$ , then we reject  $H_0$  and conclude that the event's information did have a material impact on security  $i$ 's return.

As stated in the Introduction, Eqn. (27) is not testing for a change in variance as is done in Beaver (1968) or Hertzler (1991). In their respective articles, the authors divide the squared residuals by the sample variance derived from the estimation window. Our  $\widehat{CAR}_i$  statistic captures the deviation between the mean returns from the estimation and event windows. However, we divide this estimator by its variance, which naturally forms a Wald statistic. A true test for a change in variance tests the assumption  $\sigma_{\epsilon_i}^2 = \sigma_{\epsilon_i}^2$  using an  $F$ -statistic (or equivalent non-parametric test). Our work is distinctly different and does not require the assumption of equal variances.

At the individual observation/event level, it isn't necessary to use the Wald statistic we propose in Eqn. (27), as there is no "averaging to zero," given there is only one observation/event  $i$ . However, this is rarely a solution, as one observation is rarely powerful enough to reject a false null hypothesis. Looking beyond an individual event and firm, we and the standard event study aggregate across multiple events of the same type. In the case of the standard approach, the aggregation has no power without "binning" the observations. In our case, it isn't an issue as the observations are squared.

The approach for testing the omnibus null hypothesis

$$\begin{aligned} H_0 : E(\widehat{CAR}_1) &= E(\widehat{CAR}_2) = \dots = E(\widehat{CAR}_N) = 0 \\ H_a : \text{At least one } E(\widehat{CAR}_i) &\neq 0. \end{aligned} \quad (28)$$

simultaneously based on a Wald statistic for an individual firm requires aggregating over firms

$$W = \sum_{i=1}^N W_i = \sum_{i=1}^N \frac{(\widehat{CAR}_i - 0)^2}{V(\widehat{CAR}_i)} \sim \chi^2(N) \quad , \quad (29)$$

where  $N$  represents the total number of firms  $i = 1, 2, \dots, N$  and  $W$  follows a  $\chi^2$ -distribution with  $\nu = N$  df. If  $W > \chi_{0.05}^2(N)$  for a significance level of  $\alpha = 0.05$ , then we reject  $H_0$  and conclude that the event did influence the return for at least one firm  $i$ .

Given the Wald test we propose here, it is important to note it can suffer from common problems in the field of inference. For example, there can be a missing variable bias if the event regularly occurs at the same time as another event. Furthermore, if the  $\widehat{\theta}$  is not normally distributed, or the asymptotic behavior is not achieved, then the use of the  $\chi^2$ -distribution may be flawed.

## 2.5 Implementation

To promote our more robust approach to event study analysis, and to further highlight the method, we provide a step-by-step guide to its implementation. Given a defined estimation and event window for securities  $i = 1, \dots, N$  and their associate security returns  $R_{it}$  and  $R_{i\tau}$ , a researcher would test for a material impact by

1. Computing model parameters  $\widehat{\alpha}_i$ ,  $\widehat{\beta}_i$ ,  $\widehat{\alpha}'_i$ , and  $\widehat{\beta}'_i$  based on Eqns. (7) and (8) for each firm  $i$ ,
2. Computing  $\widehat{AR}_{i\tau}$  for each firm and day in the event window following Eqn. (17) and summing across the event window to compute  $\widehat{CAR}_i$  for each firm  $i$ ,
3. Computing  $V(\widehat{CAR}_i)$  using Eqn. (24) and (likely) the mean squared error from the OLS estimation as an estimate for  $\sigma_{\epsilon_i}^2$  and  $\sigma_{\epsilon'_i}^2$  following Eqn. (15),
4. Computing  $W_i$  for an individual firm, or  $W$  for a group of firms, following Eqns. (27) and (29), respectively, and
5. Evaluating whether  $W$  for a group of firms (or  $W_i$  for an individual firm) is above or below the critical value  $\chi_{\alpha}^2(N)$  (or  $\chi_{\alpha}^2(1)$  for an individual firm) and rejecting, or failing to reject, the null hypothesis at a significance level  $\alpha$  and by extension whether the information had a material impact on security prices (or price for an individual firm).

Note, the main difference in this step-by-step process to the standard event study approach is in Step 4 where one calculates

$$W = \sum_{i=1}^N \frac{(\widehat{CAR}_i - 0)^2}{V(\widehat{CAR}_i)} \sim \chi^2(N) \quad , \quad (30)$$

instead of

$$z_0 = \frac{1}{N} \sum_{i=1}^N \frac{(\widehat{CAR}_i - 0)}{V(\widehat{CAR}_i)^{1/2}} \sim N(0, 1) \quad , \quad (31)$$

and in Step 5, where the former follows a  $\chi^2(N)$  distribution and the latter follows a standard normal distribution. In other words, the computations are nearly equivalent and therefore our approach can easily be completed by a researcher if they have already completed a standard event study.

With this comparison in mind, our method has other similar characteristics to that of the standard event study. For instance, one can easily increase or decrease the size of the event window. The benefits of increasing the size of the event window is two fold: one has additional observations and can reduce the variation in the estimate of  $\sigma_{\epsilon}^2$ , and can capture any delay in incorporating the information into the stock price. However, similarly to standard event studies, determining the length of the event window is ad-hoc and the longer the window the less power the test has in being able to reject a false null. Furthermore, longer event windows may induce some of them to overlap and violate the assumption of independence between error terms. To reiterate, many benefits and drawbacks of standard event studies are true when using the Wald test proposed here.

### 3 RESULTS & DISCUSSION

#### 3.1 Comparing the Wald, z, and K-S tests

In the event study literature, tests of the null hypothesis given in Eqn. (28) using what Campbell et al. (1997) terms the standardized cumulative abnormal return as provided in Eqn. (35) are well established. The reader will note that it is common, although unnecessary, to make the simplifying assumptions necessary to use Eqn. (25) for  $V(\widehat{CAR}_i)$ . As a result, it is trivial to switch to the Wald test we describe, since it simply requires squaring  $z$  statistics and using the  $\chi^2$  distribution.

As an added comparison, a one-sample Kolmogorov-Smirnov (KS) approach as first discussed by Falk and Levy (1989) and more recently by Larsen and Resnick (1999) will be included in our evaluation of our Wald test and the standard normal approach. Specifically, this entails testing whether  $\widehat{CAR}_i \sim N(0, 1)$  for  $i = 1, \dots, N$ .<sup>5</sup> Since this K-S approach is a non-directional test free of the binning requirement, we provide it as a comparison to the Wald statistic approach we are promoting. The K-S test can be thought of as having power without knowing how to “bin” observations.

Given a set of ordered observations  $\{Z_i\}_{i=1}^N$  where

$$Z_i = \frac{(\widehat{CAR}_i - 0)}{V(\widehat{CAR}_i)^{1/2}} \quad , \quad (32)$$

we compute the empirical cumulative distribution function (CDF)  $F_n(z)$

$$F_N(z) = \frac{1}{N} \sum_{i=1}^N I_{[-\infty, z]}(Z_i) \quad , \quad (33)$$

where the indicator function  $I_{[-\infty, z]}(Z_i) = 1$  when  $z \leq Z_i$  and zero otherwise. Given a reference CDF  $F(z)$ , which in our case is the standard normal, we compute the K-S

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<sup>5</sup>In footnote 7, it appears Hertzler (1991) uses the equivalent of the Wald statistic we are proposing and tests the null of no effect using a Kolmogorov-Smirnov test.

statistic as

$$D_N = \sup_z |F_N(z) - F(z)| .$$

We then test the K-S statistic using the critical values of the K-S distribution and reject the null hypothesis that the empirical distribution is standard normal if

$$\sqrt{N}D_N > K_\alpha , \quad (34)$$

where the critical value  $K_\alpha$  satisfies

$$Pr(K \leq K_\alpha) = 1 - \alpha . \quad (35)$$

Whether one uses the Wald,  $z$ , or K-S tests, the machinery necessary to switch from one to the other is relatively straightforward. They are all based on assuming a standard normal distribution at the firm level ( $\widehat{CAR}_i$ ). Given a set of statistics  $\{\widehat{CAR}_i\}_{i=1}^N$ , one can normalize and average them ( $z_0$ ), normalize, square, and sum them ( $W$ ), or compare them to a standard normal distribution (K-S).

### 3.2 Power

Now that the similarities and ease of switching from one to the other has been established, we evaluate and compare their relative power. Comparing each test's performance based on an analysis of power is more critical than identifying their similarities from an implementation standpoint. We will show that the Wald statistic is the optimal approach when "binning" the observations is not possible.

Power is the probability of rejecting the null hypothesis given that the alternate is true. This is defined analytically for the Wald test as

$$P(\alpha, H_a) = Pr(W > c_\chi(\alpha) | H_a) , \quad (36)$$

where  $c_\chi(x) = \chi^2(N)^{-1}(x)$  is the critical value of the  $\chi^2$ -distribution with  $\nu = N$  df. For the  $z$ -statistic, power is given by

$$P(\alpha, H_a) = Pr(z_0 < c_z\left(\frac{\alpha}{2}\right) | H_a) + Pr(z_0 > c_z\left(1 - \frac{\alpha}{2}\right) | H_a) \quad (37)$$

where the critical value  $c_z(x) = \phi^{-1}(x)$  is determined from the  $N(0, N)$  distribution. Finally, for the K-S statistic, we find

$$P(\alpha, H_a) = Pr(\sqrt{n}D_n > K_\alpha | H_a) , \quad (38)$$

where  $K_\alpha$  is the aforementioned critical value for the K-S distribution.

To tabulate and compare power for each statistical approach, one must posit plausible scenarios for the model parameters. In this work, we use those described in the benchmark article of MacKinlay (1997) by assuming abnormal returns of 0.5% and 1.5% over the event window at the event level, a cumulative variance in returns of 0.0004 with

$\sigma_{\epsilon'_i}^2 = \sigma_{\epsilon_i}^2$ , and a Type-I error rate of  $\alpha = 0.05$ .<sup>6</sup> Also, to compare the three approaches, it is important to note the  $z$ -statistic approach requires binning of observations (such as earnings announcements) as above expectation, below expectation, or at expectation.

In the case of binning, we follow MacKinlay (1997) to posit three bins are necessary, where only the bins of “good news,” or the earnings announcement beats analysts’ expectations by a specific margin, and “bad news,” where the announcements are below analysts’ by a specific margin, are used to test for an abnormal return. Furthermore, a Bonferroni correction is applied given the multiple comparisons that are made (i.e., one tests whether the “good news” sample or the “bad news” sample rejects the null hypothesis). As additional tests introduces a higher likelihood of Type I errors, the binning case requires the power for the  $z$ -statistic to be calculated as

$$P(\alpha, H_a) = \sum_{i \in \mathcal{B}} Pr(z_{0i} < c_z \left(\frac{\alpha}{2b}\right) | H_a) + Pr(z_{0i} > c_z \left(1 - \frac{\alpha}{2b}\right) | H_a) \quad (39)$$

where  $b$  is the number of bins ( $\mathcal{B}$ ), and  $z_{0i}$  is the  $z$ -statistic calculated from Eqn. (35) using observations from each bin ( $\mathcal{B}$ ). Again, following MacKinlay (1997), we assume for our power analysis that there are two bins in  $\mathcal{B}$  where one-third can be clearly labeled as “positive,” one-third as “negative,” and one-third cannot be distinguished. As a result, the  $z$  test has one-third less observations due to issues of binning accuracy. Furthermore, we have binned the observations for the K-S test as well as incorporated a Bonferroni correction, as it has improved power as will be shown below.

Based on these assumptions, we have plotted the power as a function of the number of events in each binning situation in Figure 1. Based on the posited environments, we can see the standard normal approach is the most efficient approach, then the K-S approach, then the  $\chi^2$  (or Wald) approach we are documenting here. These results assume binning is possible, i.e., market expectations are known and accurate.

The key take away is the  $z$ -statistic, i.e., the canonical event study approach, is the way to go under the assumptions of the effect of the event on the securities’ prices, variability in returns and binning feasibility.

What we are focused on is a situation where the latter is not feasible. If one assumes binning isn’t possible, or market expectations are unknown, then the  $\chi^2$  approach is a clear winner as seen in Figure 2. Coupled with the fact it is an easy extension from the  $z$ -test, we recommend it if binning isn’t possible or feared to be inaccurate.

The power analyses above are based on a presumption that the estimation window is sufficiently large so that  $\sigma_{\epsilon_i}^2$  is known and the simplifying assumptions related to the variance, or Eqn. (25), applies.

Note, if one assumes  $\sigma_{\epsilon'_i}^2$  must be estimated from the data and  $N_\tau$  is relatively small, then be cautious when using the Wald statistic we are proposing. If  $\sigma_{\epsilon'_i}^2$  is estimated inaccurately, then the results may be misleading.

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<sup>6</sup>We simulate the power assuming Eqn. (25), returns are normally distributed, and  $N_\tau = 90$  for each event.

Figure 1: Power of Tests with Binning

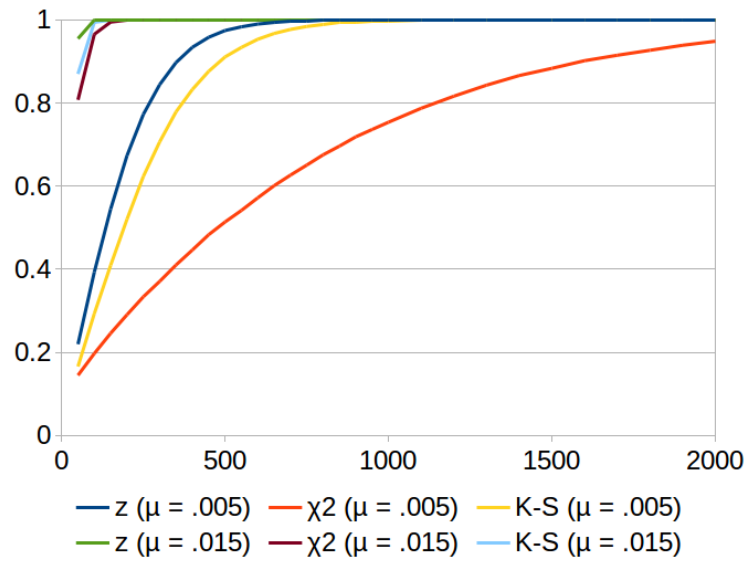
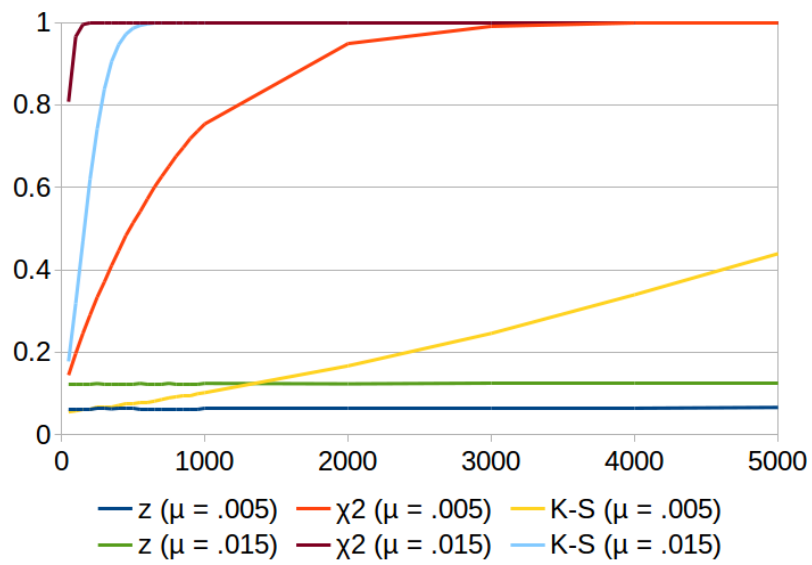


Figure 2: Power of Tests without Binning





### 3.3 Applications

We provide three examples. The first demonstrates the method in a well known example - earnings announcements. The second and third examples provide situations when the method may be useful as the market expectations are unknown and the effect of the event may be positive or negative theoretically.

#### 3.3.1 Earnings announcements

A standard example for event studies is analyzing the effect earnings announcements on a firm's stock price, e.g., MacKinlay (1997) uses it to highlight his survey of event studies.

To highlight the issue of "binning" in the standard approach, and how our Wald statistic circumvents it, we analyze the effect of a company's earnings guidance relative to the census forecast provided by the Institutional Brokers Estimate System, or I/B/E/S. The I/B/E/S breaks the guidance into three categories - "beat," "matched," and "missed" relative to market expectations.

To reiterate, the theoretical issue is if one attempts to analyze the effect of a company's guidance on its stock price, without knowing market expectations, then one could find no effect. It is due to the fact some companies may beat market expectations, and thus likely increasing their price, while others may miss market expectations, thereby lowering their stock price. Without market expectations, and splitting out the different types of announcements, the standard event study approach would aggregate both of these effects into one test statistic and they could wash out the underlying impact of announcements overall.

The canceling effect can be seen in Table 1 where we estimate the effect of earnings announcements on stock prices. Specifically, if the data are pooled (not binned), then the  $z$ -statistic fails to reject a hypothesis that earnings guidance has no effect on stock prices. Put differently, a researcher would fail to find an effect of announcements on earnings. However, the Wald statistic (and KS approach) with the pooled data can reject the null hypothesis of no effect.

Now, in this example, we have market expectations. Using I/B/E/S's definition of the types of announcements (beat, missed, matched), we can separate the sample and find an effect using the standard event study approach ( $z$ -statistic). However, the Wald statistic is a simple extension of the standard approach.

Market expectations, and by extension binning, can contain measurement error. One can see it in the current example, or Table 1. Specifically, the standard approach after binning fails to reject no effect for firms who "match market expectations." However, the Wald and KS approaches still reject even though firms match the market as defined by I/B/E/S. In other words, firms who meet market expectations can still affect their stock price in this example. It is something that can be missed with the standard event study approach.

### 3.3.2 Olympic announcements

In this second example, we apply the Wald statistic to the analysis of whether a country winning or losing the chance to host the Olympics, as announced by the International Olympics Committee on a particular day, has an effect on a country's stock market.

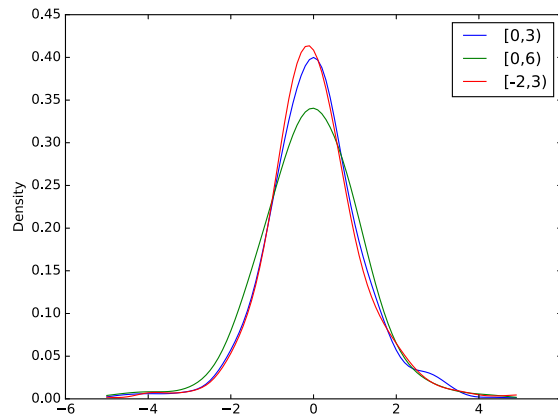
In general, there has been a debate of the benefits of hosting a sporting event such as the Olympic Games (e.g., Baade and Matheson (2016)). An argument can be made hosting the Olympics requires burdensome public funding outlays and can do more harm than good. Alternatively, others suggest there are huge gains that support the use of public funding.

In terms of event studies related to Olympic announcements, there has been a debate on whether the announcement has an impact as discussed in Dick and Wang (2010) and Engelhardt et al. (2018b). As the effect may be positive or negative, and there isn't a clear indicator of what the market expects, the situation is a good place to apply the Wald statistic. Furthermore, neither papers analyze the potential for a "mixing" of positive or negative announcements in their analysis.

The results of the analysis on Olympic bids come from the data available in Tables 3 and 4 of Engelhardt et al. (2018b) where the primary losers and winners are used in the analysis. The data has been replicated in Table 2. The full dataset is provided in this example to allow individuals to replicate the tests.

The primary studies on the impact of the announcements separate the winning and losing bids. The separation is the same as "binning." The results by binned groups are provided in Table 3. In this case, the Wald statistic does not add to the previous literature. However, the research focuses on splitting the groups into winners and losers. With the Wald approach, the focus can be changed away from assuming one group is "better off." In particular, given the research is unclear on whether it is a good or bad for a country to win or lose their bid, winning or losing might not put countries in an "good" or "bad" bin.

The results from testing a null hypothesis of a zero mean using all the observations are in Table 4. We provide the results from the  $z$ , Wald, and KS tests. As one can see, the results still fail to reject the null hypothesis that the abnormal return for countries winning and losing the bid, at the time of the announcement, is zero. This is true for all three types of tests. Nonetheless, the application demonstrates one does not need to know whether the announcement was above or below market expectations. The application here is particularly poignant as there may be a mixed outcome - winning or losing could both be good or bad depending upon whether the country won or lost, the country's characteristics, and potentially changing information about whether hosting the Olympics has positive or negative outcomes for a host country. In other words, it isn't clear a country winning the bid is a good thing or good from one point in time to the next. It may depend. As a result, the Wald statistic is potentially necessary to test for a material impact from the announcement.

Figure 3: Smoothed Histogram of Patent Event  $z_{0s}$ 

### 3.3.3 Patent decisions

In our final example, we apply the Wald statistic to the analysis of whether the outcome of an appealed United States patent decision, as decided by the United States Court of Appeals for the Federal Circuit, has an effect on a firm's stock price.

The results here are reiterating the more detailed work of Engelhardt et al. (2018a). As a summary, patent decisions can transfer substantial monetary values between a defendant and plaintiff. As a result, the publicly traded firms involved in these cases can see their stock price rise or fall. However, binning the observations by “winner” and “loser” is extremely difficult as it is relative to market expectations. In other words, Company A may be awarded \$1 billion as part of a court case, but this might be \$1 billion less than the market expected. Therefore, what appears to a naive econometrician is company A winning in the appeal. However, Company A's stock price will fall as the market expected more.

The court case dates were collected from Docket Navigator and the stock data from Yahoo Finance. Given the significantly larger data set (roughly 687 observations depending upon event window size), we plot the standardized cumulative abnormal returns in Figure 3. The statistics to test the null of no effect are provided in Table 5.

As one can see, if a researcher simply used the  $z$  or KS test, then they would fail to realize patent decisions are materially important. However, if the Wald statistic is applied, which has been shown to be more powerful in Section 3.1, we are able to reject the null hypothesis of no announcement effect.

## 4 CONCLUSION

In event studies where market expectations are undefined, we provide a squared test statistic, i.e., a Wald statistic, to test for a material impact of an announced outcome.

To demonstrate its usefulness, we have shown in Section 3.2 its improved power over both the standard event study approach and a K-S approach. Furthermore, in Section 3.3, we showed its importance in demonstrating the material impact of patent decisions and its potential applicability in other situations. For example, does a political election, the retirement of a CEO, greenhouse gas emission standards, or tariffs impact companies' bottom lines? As the stock price will move relative to market expectations, or as these outcomes may help some firms and hurt others, one cannot test for their impact with a standard event study approach without market expectations. Our approach alleviates the need to know expectations and it is a simple addition to the standard approach.

What we propose is not without some drawbacks. For instance, if there is a good estimate of market expectations regarding the event, e.g., it was above or below analyst forecasts, then the standard event study approach is more powerful as demonstrated in Section 3.2. Furthermore, to estimate the idiosyncratic variation in returns in the event window ( $\sigma_{\varepsilon}^2$ ), one must either assume its variation is equal to the equivalent variation in the estimation window ( $\sigma_{\varepsilon}^2$ ) or use the sample variation in the event window to estimate it. In the later case, the size of the event window can make the estimate large and inhibit our proposed approach. However, the standard event study uses the variation from the estimation window and making the same assumption here eliminates the issue. Finally, our approach assumes the idiosyncratic variation is normally distributed or the Wald statistic achieves asymptotic behavior ( $\chi^2$ -distributed). If these are not true, then a two-sample K-S test similar to the one discussed in Section 3.1 or other alternative may be more appropriate. However, again, the standard event study approach assumes the variation is normally distributed and the non-parametric approaches often, although not always (Campbell and Wesley (1993)), provide similar results. However, extension of the current work regarding the relaxation of the normal distribution assumption or testing for sufficient sample sizes may provide more insight.

To summarize, we recommend researchers add to their event study analysis the use of a Wald test to ensure against unknown or poorly measured market expectations. It is a simple and straightforward extension when using the standard event study approach.

## Conflict of Interest

The authors confirm that this article content has no conflict of interest.

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Table 1: Test of Earnings Guidance on Stock Prices

	Event windows		
	[0, 3)	[0, 6)	[-2, 3)
<u>Pooled Data</u>			
z statistic	-1.451	-1.71	-0.958
p-value with $z_0$	0.147	0.087	0.338
$W$	3733.726	2699.843	2355.292
p-value with $W$	0.0	0.0	0.0
KS statistic	0.174	0.155	0.139
p-value with KS	0.0	0.0	0.0
<u>Beat Market Expectations</u>			
z statistic	8.385	6.283	6.098
p-value with $z_0$	0.0	0.0	0.0
$W$	466.886	373.87	286.598
p-value with $W$	0.0	0.0	0.0
KS statistic	0.372	0.297	0.305
p-value with KS	0.0	0.0	0.0
<u>Missed Market Expectations</u>			
z statistic	-11.537	-8.982	-8.372
p-value with $z_0$	0.0	0.0	0.0
$W$	909.557	579.101	563.697
p-value with $W$	0.0	0.0	0.0
KS statistic	0.414	0.404	0.388
p-value with KS	0.0	0.0	0.0
<u>Matched Market Expectations</u>			
z statistic	-0.619	-1.088	-0.339
p-value with $z_0$	0.536	0.276	0.734
$W$	2354.623	1742.897	1503.879
p-value with $W$	0.0	0.0	0.0
KS statistic	0.158	0.132	0.129
p-value with KS	0.0	0.0	0.0

Note: The data is based on companies who provide annual guidance numbers as point estimates and per share between Jan 2016 to Dec 2017 and available in I/B/E/S. The data contains 63 observations above market expectations, 59 below market expectations, and 378 matching expectations.

Table 2:  $z_0$  by Bid

Olympic Year	Season	Country	Announcement	Bid	$z_0$ for event windows		
					[0, 3)	[0, 6)	[-2, 3)
1988	Summer	Japan	Sep 30, 1981	Lost	-0.01	0.93	-0.48
1988	Summer	South Korea	Sep 30, 1981	Won	-1.25	1.58	-0.47
1988	Winter	Sweden	Sep 30, 1981	Lost	1.07	1.3	-1.13
1988	Winter	Canada	Sep 30, 1981	Won	0.39	-0.42	3.47
1992	Summer	France	Oct 17, 1986	Lost	-1.72	0.19	-2.16
1992	Summer	Spain	Oct 17, 1986	Won	-0.93	-1.56	-1.08
1992	Winter	France	Oct 17, 1986	Won	-1.72	0.19	-2.16
1994	Winter	Sweden	Sep 15, 1988	Lost	-0.24	0.95	0.17
1994	Winter	Norway	Sep 15, 1988	Won	0.92	0.82	0.95
1996	Summer	Greece	Sep 18, 1990	Lost	-1.67	-2.48	-0.09
1996	Summer	USA	Sep 18, 1990	Won	0.77	0.21	0.27
1998	Winter	USA	Jun 15, 1991	Lost	-0.22	-0.73	0.1
1998	Winter	Japan	Jun 15, 1991	Won	0.13	-0.34	0.09
2000	Summer	China	Sep 23, 1993	Lost	-0.13	0	0.08
2000	Summer	Australia	Sep 23, 1993	Won	1.13	0.87	0.35
2002	Winter	Switzerland	Jun 16, 1995	Lost	-0.13	0.35	-0.12
2002	Winter	Sweden	Jun 16, 1995	Lost	0.98	0.97	0.11
2002	Winter	USA	Jun 16, 1995	Won	1.51	0.86	0.79
2004	Summer	Italy	Sep 05, 1997	Lost	0.28	-0.73	-0.47
2004	Summer	Greece	Sep 05, 1997	Won	2.98	1.42	1.92
2006	Winter	Switzerland	Jun 19, 1999	Lost	0.04	-0.51	0
2006	Winter	Italy	Jun 19, 1999	Won	-0.1	-0.06	0.16
2008	Summer	Canada	Jul 13, 2001	Lost	-0.1	0.32	-0.29
2008	Summer	China	Jul 13, 2001	Won	-0.76	0.23	-1.17
2010	Winter	South Korea	Jul 02, 2003	Lost	0.59	0.93	0.59
2010	Winter	Canada	Jul 02, 2003	Won	-0.6	0.31	-0.26
2012	Summer	France	Jul 06, 2005	Lost	-0.38	0.55	0.07
2012	Summer	UK	Jul 06, 2005	Won	-0.51	-0.02	0.8
2014	Winter	South Korea	Jul 04, 2007	Lost	2.03	2.28	2.86
2014	Winter	Russia	Jul 04, 2007	Won	0.62	0.6	0.55
2016	Summer	Spain	Oct 02, 2009	Lost	0.27	-0.31	-0.03
2016	Summer	Brazil	Oct 02, 2009	Won	1.24	0.3	0.54
2018	Winter	Germany	Jul 06, 2011	Lost	-0.07	-0.37	-0.17
2018	Winter	South Korea	Jul 06, 2011	Won	0.47	-0.54	1.24
2020	Summer	Turkey	Sep 07, 2013	Lost	2.34	2	1.41
2020	Summer	Japan	Sep 07, 2013	Won	1.03	0.12	-0.06
2022	Winter	China	Jul 31, 2015	Won	-1.29	-0.3	0.12

Note: A detailed discussion of the data, and its calculation, can be found in Engelhardt et al. (2018b). The statistics are tabulated in the same fashion describe here, or in particular following Equations 17, 21, 25, and 35.



Table 3: Test of Olympic Bids Affecting Host Country Stock Market by Type

	Event windows		
	[0, 3)	[0, 6)	[-2, 3)
<u>Winning Bids</u>			
z statistic	0.928	0.985	1.396
p-value with $z_0$	0.353	0.325	0.163
$W$	25.508	10.509	27.792
p-value with $W$	0.145	0.939	0.088
KS statistic	0.177	0.243	0.221
p-value with KS	0.544	0.178	0.268
<u>Losing Bids</u>			
z statistic	0.695	1.332	0.111
p-value with $z_0$	0.487	0.183	0.912
$W$	18.266	22.754	17.122
p-value with $W$	0.438	0.2	0.515
KS statistic	0.24	0.213	0.265
p-value with KS	0.213	0.342	0.132

Table 4: Test of Olympic Bids Affecting Host Country Stock Market: Winners and Losers

	Event windows		
	[0, 3)	[0, 6)	[-2, 3)
z statistic	1.149	1.635	1.078
p-value with $z_0$	0.25	0.102	0.281
$W$	43.774	33.263	44.914
p-value with $W$	0.206	0.645	0.174
KS statistic	0.137	0.188	0.181
p-value with KS	0.461	0.128	0.155

Table 5: Test of Patent Decisions Affecting Firm's Stock Price

	Event windows		
	[0, 3)	[0, 6)	[-2, 3)
z statistic	0.549	0.193	0.643
p-value with $z_0$	0.292	0.423	0.26
$W$	1120.041	2060.189	1100.046
p-value with $W$	0.0	0.0	0.0
KS statistic	0.037	0.04	0.04
p-value with KS	0.316	0.233	0.235